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Lorentz harmonics and superfield action. $D = 10,\ N = 1$ superstring

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Abstract

We propose a new version of the superfield action for a closed $D = 10,\ N = 1$ superstring where the Lorentz harmonics are used as auxiliary superfields. The incorporation of Lorentz harmonics into the superfield action makes possible to obtain superfield constraints of the induced worldsheet supergravity as equations of motion. Moreover, it becomes evident that a so-called 'Wess-Zumino part' of the superfield action is basically a Lagrangian form of the generalized action principle. We propose to use the second Noether theorem to handle the essential terms in the transformation lows of hidden gauge symmetries, which remove dynamical degrees of freedom from the Lagrange multiplier superfield.

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Introduction

Recently a new interest to the superfield description of superbranes is witnessed [1, 2]. The superfield actions for superbranes might be useful in a search for new superconformal theories [3].

The superfield description of the Brink-Schwarz superparticle was discovered by Sorokin, Tkach, Volkov and Zheltukhin (STVZ) in Refs. [4, 5, 6] (see [7] for a nice review). In [4] and [6] the geometrical origin of κ-symmetry as local worldline supersymmetry has been established for $D = 3, 4$ and $D = 10$ superparticles. The STV–like actions have been also constructed for superstrings and some superbranes with not more than 16 target space supersymmetries and 8 worldvolume supersymmetries ([8]–[17] and Refs. in [7]). So, the action for a $D = 10$, $N = 1$ closed superstring (heterotic superstring without heterotic fermions) was built in [12] [1]. The purpose of this paper is to present another form of the action for such an object where Lorentz harmonics [21, 22, 23, 24, 25] (see also [26, 27, 28]) are used.

The main advantage of this action is that the constraints of $d = 2$, $n = (8, 0)$ induced supergravity can be derived as equations of motion, while in the original approach [4, 11, 12, 7] these constraints are imposed 'by hands'. Such property might be useful for the consideration of STV-like actions for higher superbranes. Moreover, the construction of such an action can be regarded as a completion of the program of passing from the component (Green–Schwarz) action to the superfield action through an intermediate step of the generalized action principle [29] (see [30] for general consideration).

The superfield action involves two Lagrange multiplier superfields. To convince oneself that they do not carry dynamical degrees of freedom, one should find some hidden gauge symmetries. Usually this is a nontrivial task. However, here we propose a shortcut, namely, we shall demonstrate how the second Noether theorem provides the possibility of finding the basic, essential terms in the transformation lows of the hidden gauge symmetries by studying an interdependence of equations of motion.

We hope that the methods of the present paper will be useful in the study of superfield formulations of higher superbranes in relatively low dimensions D=4,5,6 as well as for the investigation of STV–like actions with 16 supersymmetries in $D = 11$ and $D = 10$ type II superspaces [14, 15] with the aim to clarify the field content of corresponding 'spinning superbrane' models [2].

The paper is organized as follows. Section 1 is devoted to a brief review of the STVZ approach to superparticles and $N = 1$ superstrings. In Section 2 we recall the generalized action principle for $D = 10$, $N = 1$ superstring [29], derive the superfield equations of motion and present some basic relations which are useful for the consideration of the superfield action. The new version of the superfield action for the $D = 10$, $N = 1$ superstring is presented in Section 3. The generation of the worldvolume supergravity constraints by superfield equations of motion which follow from the superfield action is the subject of Section 4. The superfield equations are investigated in Section 5. Then,

[1] The problem of superfield description of the heterotic fermions is the separate subject considered in Refs. [18, 19, 20].

[2] We call such dynamical systems 'spinning superbranes' as i) they have both the worldsheet and space–time supersymmetry manifest and ii) they contain additional (in comparison with the Green–Schwarz superstring and $D = 11$ supermembrane [31]) dynamical degrees of freedom [25]. Due to these properties they can be regarded as some extended counterparts of the 'spinning superparticle' models [32].

in Section 6, we use the second Noether theorem to find the gauge symmetries of the superfield action, including hidden gauge symmetries which act on the Lagrange multiplier superfields. These symmetries are used in Section 7 to prove the pure auxiliary nature of the Lagrange multiplier superfields and to derive the component equations of motion. Some technical details are collected in Appendix.

1 STVZ approach to superparticle and $N=1$ superstring. Brief review.

The STVZ approach to the description of the Brink-Schwarz superparticle [4, 6, 7] is based on the consideration of an embedding of the world sheet superspace

$$\mathcal{M}^{(1|n)} = (\tau, \eta^q) \equiv (\tau^{++}, \eta^{+q}), \qquad q = 1, \dots n \tag{1}$$

into the D-dimensional target superspace ($D = 3, 4, 6, 10$, $N = 1$)

$$\underline{\mathcal{M}}^{(D|2(D-2))} = (X^{\underline{m}}, \Theta^{\underline{\mu}}), \tag{2}$$

$$\underline{m} = 0,\ 1, \dots, (D-1) \qquad \underline{\mu} = 0,\ 1, \dots, 2(D-2).$$

In (1) the sign indices $++$, $+$ of the worldline coordinates τ^{++}, η^{+q} denote their scaling dimension (see below). Their usage is instructive in some places (see e.g. Eq.(7)).

The embedding

$$\mathcal{M}^{(1|n)} \rightarrow \underline{\mathcal{M}}^{(D|2(D-2))} \tag{3}$$

can be defined locally by coordinate superfields

$$X^{\underline{m}} = \hat{X}^{\underline{m}}(\tau^{++},\ \eta^{+q}) = \hat{x}^{\underline{m}}(\tau) + \eta^{+q}\ \chi^{\underline{m}}_{+q}(\tau) + \dots + (\eta^{+q})^n\ S^{\underline{m}}_{[+n]}(\tau), \tag{4}$$

$$\Theta^{\underline{\mu}} = \hat{\Theta}^{\underline{\mu}}(\tau^{++}, \eta^{+q}) = \hat{\theta}^{\underline{\mu}}(\tau) + \eta^{+q}\ \lambda^{\underline{\mu}I}_{+q}(\tau) + \dots + (\eta^{+q})^n \Sigma^{\underline{\mu}I}_{[+n]}(\tau).$$

Here and in what follows we use the compact notations for higher degrees of the Grassmann variables

$$(\eta)^n \equiv \frac{1}{n!}\epsilon_{q_1 \dots q_n}\eta^{+q_1} \dots \eta^{+q_n}, \qquad (\eta)^{n-1}_q \equiv \frac{1}{(n-1)!}\epsilon_{q q_1 \dots q_{n-1}}\eta^{+q_1} \dots \eta^{+q_{n-1}}. \tag{5}$$

The most complete description, where all κ–symmetries are replaced by worldsheet supersymmetries, is achieved when the number of worldvolume Grassmann coordinates η^{+q} is half the number of target space Grassmann coordinates $\Theta^{\underline{\mu}}$ (for $D = 3, 4, 6, 10$, $N = 1$, $\underline{\mu} = 1, 2, \dots, 2(D-2)$, $q = 1, 2, \dots, (D-2)$).

To describe the Brink–Schwarz superparticle (but not the so–called spinning superparticle model [32] containing additional degrees of freedom) the embedding (3) should be subject to the constraints

$$\hat{\Pi}^{\underline{m}}_{+q} = D_{+q}\hat{X}^{\underline{m}}(\tau, \eta) - iD_{+q}\hat{\Theta}^{\underline{\mu}I}\Gamma^{\underline{m}}_{\underline{\mu}\underline{\nu}}\ \hat{\Theta}^{\underline{\nu}I}(\tau, \eta) = 0, \tag{6}$$

where

$$D_{+q} = \partial_{+q} + 2i\eta^{+q}\partial_{++}, \qquad \partial_{++} \equiv \frac{\partial}{\partial \tau^{++}}, \qquad \partial_{+q} \equiv \frac{\partial}{\partial \eta^{+q}} \tag{7}$$

and $\Gamma^{\underline{\mu}}_{\underline{\mu}\underline{\nu}} = \Gamma^{\underline{\mu}}_{\underline{\nu}\underline{\mu}}$ are 10-dimensional 16×16 γ-matrices in Majorana-Weyl representation. Eq.(6) was called the geometrodynamic condition [11] or the basic superembedding equation [33, 7]. It implies the vanishing of the fermionic components of the pull-back (on the worldline superspace)

$$\hat{\Pi}^{\underline{m}} = d\hat{X}^{\underline{m}} - i\ d\hat{\Theta}^{\underline{\mu}I}\Gamma^{\underline{m}}_{\underline{\mu}\underline{\nu}}\ \hat{\Theta}^{\underline{\nu}I} = w^{++}\hat{\Pi}^{\underline{m}}_{++} + d\eta^{+q}\hat{\Pi}^{\underline{m}}_{+q} \tag{8}$$

of the covariant bosonic form (vielbein) of the flat target superspace

$$\Pi^{\underline{m}} = dX^{\underline{m}} - id\Theta^I\Gamma^{\underline{m}}\Theta^I. \tag{9}$$

In Eq.(8) the 1–forms

$$w^{++} = d\tau^{++} - 2id\eta^{+q}\eta^{+q}, \qquad d\eta^{+q} \tag{10}$$

provide a basis (supervielbein) for the differential forms on the worldline superspace. This basis is invariant under superconformal symmetry [23, 11]

$$\delta\tau^{++} = \Lambda^{++}(\tau,\eta) = 2a^{++}(\tau) + 4i\eta^{+q}\kappa^{+q}(\tau) + \frac{1}{2}\eta^{+q}\eta^{+p}\ a_{[qp]}(\tau) + \ldots, \tag{11}$$

$$\delta\ \eta^{+q} = -\frac{i}{4}D_{+q}\Lambda^{++}(\tau,\eta) = \kappa^{+q} + \eta^{+p}\ (\delta_{qp}\partial_{++}a^{++}(\tau) + a_{qp}(\tau)) + \ldots.$$

The components $a^{++}(\tau)$, $\kappa^{+q}(\tau)$, $a_{pq}(\tau) = -a_{qp}(\tau), \ldots$ of the superfield $\Lambda^{++}(\tau,\eta)$ can be regarded as the parameters of gauge symmetries of the STV action [4, 11]

$$S_0 = \int d\tau \hat{d}^n\eta\ P_{\underline{m}q}\ \Pi^{\underline{m}}_{+q}, \tag{12}$$

This action is basically the geometrodynamic condition (6) incorporated with the Lagrange multiplier superfield $P_{\underline{m}}(\tau,\eta)$ whose weight under the induced Weyl rescaling with the parameter $\frac{1}{2}\partial_{++}\Lambda^{++}(\tau,\eta)$ (scaling dimension) is equal to $[+(n-1)]$. (The scaling dimension of the superspace measure $d\tau\hat{d}^n\eta$ is $[1-n]$).

When the geometrodynamic condition Eq.(6) has no dynamical equations among its consequences, the Lagrange multiplier $P_{\underline{m}q}(\tau,\eta)$ does not contain superfluous dynamical degrees of freedom[3]. After making use of the equations of motion

$$D_{+q}P_{\underline{m}q}(\tau,\eta) = 0, \qquad P_{\underline{m}q}(\tau,\eta)\ D_{+q}\hat{\Theta}^{\underline{\mu}I}(\tau,\eta)\ \Gamma^{\underline{m}}_{\underline{\mu}\underline{\nu}} = 0, \tag{13}$$

and an infinitely reducible gauge symmetry [11] [4] it can be reduced to the form

$$P_{\underline{m}q} = \frac{1}{(n-1)!}\epsilon_{qq_1\ldots q_{n-1}}\eta^{+q_1}\ldots\eta^{+q_{n-1}}\ p_{\underline{m}} \equiv (\eta)^{n-1}_q p_{\underline{m}} \tag{14}$$

where $p_{\underline{m}}$ is a light–like constant vector [5] which can be identified with the momentum of a massless superparticle. For the cases when the geometrodynamic condition contains equations of motion among its consequences, as it is for a $D = 10$ type II ($N = 2$) and for a $D = 6$ type IIB superparticle [17], the Lagrange multiplier superfield contains

[3] This happens for $D = 3,4,6,10$, $N = 1$ and $D = 3,4,6(IIA)$, $N = 2$ superparticle, see [13, 17]

[4] This gauge symmetry acts on the Lagrange multiplier superfield only $\delta P_{\underline{m}q} = D_{+p}(\Sigma^{qpr}\Gamma_{\underline{m}}D_{+r}\hat{\Theta})$. Its superfield parameter is symmetric and traceless $\Sigma^{pqr\underline{\mu}}(\tau,\eta) = \Sigma^{(pqr)\underline{\mu}}$, $\Sigma^{ppr\underline{\mu}} = 0$.

[5] Indeed, due to Eq.(13) $\partial_{++}p_{\underline{m}}(\tau) = 0 \quad \Rightarrow \quad p_{\underline{m}} = const, \quad p_{\underline{m}}p^{\underline{m}} = 0$.

additional dynamical degrees of freedom. Thus the action Eq. (12) describes a spinning superparticle model [32] in these cases [6].

The STV action (12) has been generalized for some branes with not more than 16 target space supersymmetries and thus not more than 8 worldsheet supersymmetries (κ-symmetries) [12, 8, 17, 34, 16, 7]. (So the barrier of 16 supersymmetries, known from the attempts to construct superfield actions for supersymmetric field theories in terms of unconstrained superfields [35], has not been surmounted yet [7]).

The superfield action for the branes has an additional term which was called the Wess-Zumino term in [12]. For $N=1$, $D=3,4,6,10$ superstring with worldsheet superspace $\mathcal{M}^{(2|(D-2))} = (\xi^{++}, \xi^{--}, \eta^{+q}) = (\zeta^M)$ ($q=1\ldots(D-2)$, $\xi^{\pm\pm} = \tau \pm \sigma$) and coordinate superfields

$$X^{\underline{m}} = \hat{X}^{\underline{m}}(\xi^{++}, \xi^{--}, \eta^{+q}), \qquad \Theta^{\underline{\mu}} = \hat{\Theta}^{\underline{\mu}}(\xi^{++}, \xi^{--}, \eta^{+q})$$

the action of Ref. [12] reads

$$S_1 = \int d^2\xi \ \hat{d}^{D-2}\eta \ P_{\underline{m}q} \Pi^{\underline{m}}_{+q} + \int d^2\xi \ \hat{d}^{D-2}\eta \ \mathcal{P}^{MN} (\hat{\tilde{\mathcal{L}}}_2 - d\ Y_1)_{NM}, \tag{15}$$

The second term contains the Lagrange multiplier superfield $\mathcal{P}^{MN} = -(-1)^{NM}\mathcal{P}^{NM}$, the auxiliary worldsheet superfield $Y_M(\zeta^N)$ ($Y_1 = d\zeta^M \ Y_M(\zeta)$) and the 2–form $\hat{\tilde{\mathcal{L}}}_2 = \frac{1}{2} d\zeta^M \wedge d\zeta^M \ \tilde{\mathcal{L}}_{MN}$; ($\tilde{\mathcal{L}}_{MN} = -(-)^{MN} \tilde{\mathcal{L}}_{NM}$), which we call Lagrangian form for a reason which became transparent below. The latter is constructed from $\hat{X}^{\underline{m}}$, $\hat{\Theta}^{\underline{\mu}}$ and some auxiliary superfield. In flat target superspace the Lagrangian form of Ref. [12] is essentially

$$\hat{\tilde{\mathcal{L}}} = \hat{B}_2 + e^{++} \wedge e^{--} \Pi^{\underline{m}}_{++} \Pi_{\underline{m}--} - dY, \tag{16}$$

where $B_2 = -i\Pi^{\underline{m}} \wedge d\Theta\Gamma^{\underline{m}}\Theta$ is the flat superspace value of the NS-NS 2-form and e^{++}, e^{--} are bosonic supervielbein forms of the worldsheet supergravity, which are either subject to some set of torsion constraints [16] or constructed from some prepotentials [12]. Remember that $\Pi^{\underline{m}}_{\pm\pm} = e^M_{\pm\pm} \Pi^{\underline{m}}_M$ and thus the action depends on the inverse supervielbein components $e^M_{\pm\pm}$ as well.

It is important that the Lagrangian form (16) is closed on the surface of the geometrodynamic equation $\Pi^{\underline{n}}_{+q} = 0$ (this property was called Weyl triviality in [10])

$$d\hat{\tilde{\mathcal{L}}}_2|_{\Pi^{\underline{n}}_{+q}=0} = 0 \tag{17}$$

Due to this property the equations of motion

$$\delta S / \delta \mathcal{P}^{MN} = 0 \qquad \Rightarrow \qquad \hat{\tilde{\mathcal{L}}}_2 = dY_1 \tag{18}$$

can be regarded as *nondynamical* equations for Y_1 [8]. Then the equations of motion

$$\delta S / \delta Y^M = 0 \qquad \Rightarrow \qquad (-)^N \partial_N \ \mathcal{P}^{NM} = 0$$

[6] An alternative approach, which was suggested in [9], consists in solving Eq. (6) in terms of unconstrained superfields ("prepotentials"). Then an action for these prepotentials can be constructed. The recent superfield formulations of gauge-fixed brane actions [1, 2] follow mainly this line.

[7] See [36, 37] for recent progress in an off-shell description of $D=10,11$ supergravity by relaxing the torsion constraints.

[8] Indeed, as $\delta Y_1 = dY_0$ is a gauge symmetry of the model, Eq.(18) completely determines Y_1 in terms of closed form $\tilde{\mathcal{L}}_2$: after gauge fixing the solution does not involve *any* indefinite constants. This means that the field $Y_1 = d\xi^M Y_M$ has no independent degrees of freedom on the mass shell.

and a set of gauge symmetries provide us with the possibility of fixing a gauge where the Lagrange multiplier $\mathcal{P}^{NM}$ acquires the form

$$\mathcal{P}^{NM} = \frac{1}{(D-2)!}\epsilon_{q_1 \ldots q_{(D-2)}}\eta^{+q_1} \ldots \eta^{+q_{(D-2)}}T \equiv (\eta)^{D-2}T, \qquad \partial_{++}T = 0. \tag{19}$$

The constant T has the meaning of a superstring tension.

The second Lagrange multiplier $P_{\underline{m}q}$ does not contain additional degrees of freedom. This can be motivated starting from the observation that the geometrodynamic condition

$$\hat{\Pi}^{\underline{m}}_{+q} = \nabla_{+q}\ \hat{X}^{\underline{m}} - i\ \nabla_{+q}\ \hat{\Theta}\ \Gamma^{\underline{m}}\hat{\Theta} = 0 \tag{20}$$

has no dynamical consequences. Thus it can be regarded as a pure algebraic equation. The Lagrange multiplier introduced to incorporate an algebraic equation, as it is intuitively clear, should not carry dynamical degrees of freedom [20].

In Eqs.(15), (20) we should use the covariant derivatives of $d=2$ supergravity $\nabla_{+q} = e^{M}_{+q}\partial_M$, $\nabla_{\pm\pm} = e^{M}_{\pm\pm}\partial_M$, but not the flat ones $D_{+q} = \partial_{+q} + 2\ i\eta^{+q}\partial_{++},\ \partial_{++},\ \partial_{--}$. Thus it is necessary either to impose the constraints of $d=2$ supergravity or use their solution. This is not a serious problem for the $N=1$, $D=10$ superstring, because $n=(8,0)$ $d=2$ supergravity is conformally flat and does not contain dynamical degrees of freedom. However, it might be a problem for the construction of the STV–like actions for other branes with $p>1$ and $n \leq 8$ worldline supersymmetries.

Here we present a reformulation of the model (15) where the supergravity constraints occur as consequences of equations of motion. It differs from (15) by the choice of the 2-form $\mathcal{L}_2$. Instead of the components of the worldsheet bosonic supervielbein forms $e^{\pm\pm}_{M}(\zeta)$ and $e^{M}_{\pm\pm}(\zeta)$ we use auxiliary light–like 10–vector superfields $U^{++}_{\underline{m}}$, $U^{--}_{\underline{m}}$ ($U^{++}_{\underline{m}}U^{\underline{m}++} = 0, U^{--}_{\underline{m}}U^{\underline{m}--} = 0$)), normalized by $U^{++}_{\underline{m}}U^{\underline{m}--} = 2$, which can be regarded as Lorentz harmonics [21]. Moreover, our form $\hat{\mathcal{L}}_2$ is nothing but *the Lagrangian form of the generalized action* [29].

In the next section we recall main properties of the generalized action and derive some formulae required for the development of the superfield formalism.

2 Generalized action and superfield description of $N=1$, $D=10$ closed superstring

The generalized action [29] for the $N=1$ $D=10$ superstring

$$S = \int_{\mathcal{M}^2} \hat{\mathcal{L}}_2 \tag{21}$$

is an integral of a Lagrangian 2−form $\mathcal{L}_2$ over an arbitrary bosonic surface $\mathcal{M}^2 = (\xi^{--}, \xi^{++}, \eta^{+q}(\xi))$ in the worldsheet superspace $\Sigma^{(2|8)} = (\xi^{--}, \xi^{++}, \eta^{+q})$. The Lagrangian 2−form

$$\mathcal{L}_2 = \frac{1}{2}\ E^{++} \wedge E^{--} - i\ \Pi^{\underline{m}} \wedge d\Theta\ \Gamma_{\underline{m}}\Theta \tag{22}$$

is the sum of the traditional Wess-Zumino 2−form $B_2 = -i\ \Pi^{\underline{m}} \wedge d\Theta\ \Gamma_{\underline{m}}\Theta$ and a kinetic term constructed with the use of vector Lorentz harmonics $U^{++}_{\underline{m}}, U^{--}_{\underline{m}}$ [21]

$$E^{\pm\pm} = \Pi^{\underline{m}}U^{\pm\pm}_{\underline{m}}, \qquad \Pi^{\underline{m}} = dX^{\underline{m}} - i\ d\Theta\Gamma_{\underline{m}}\Theta, \tag{23}$$

2.1 $SO(1,9)/SO(1,1)\times SO(8)$ Lorentz harmonics

The Lorentz harmonics $U^{\pm\pm}_{\underline{m}}$ are elements of the Lorentz group valued matrix

$$U_{\underline{m}}{}^{\underline{a}} = (\frac{1}{2}(U^{++}_{\underline{m}} + U^{--}_{\underline{m}})),\ U^{I}_{\underline{m}}, \frac{1}{2}(U^{++}_{\underline{m}} - U^{--}_{\underline{m}})) \in SO(1,9) \quad \Leftrightarrow \quad U^{\underline{a}}_{\underline{m}}\eta^{\underline{mn}}U^{\underline{b}}_{\underline{n}} = 0 \tag{24}$$

It can be used to change the basis of cotangent superspace from $(\Pi^{\underline{m}}, d\Theta^{\underline{\mu}})$ to

$$E^{\underline{A}} = (E^{\underline{a}}, E^{\underline{\alpha}}) \tag{25}$$

where

$$E^{\underline{a}} = \left(\frac{1}{2}(E^{++} + E^{--}),\ E^{I},\ \frac{1}{2}(E^{++} - E^{--})\right) = \Pi^{\underline{m}}U^{\underline{a}}_{\underline{m}}, \tag{26}$$

$$E^{\pm\pm} = \Pi^{\underline{m}}U^{\pm\pm}_{\underline{m}}, \qquad E^{I} = \Pi^{\underline{m}}U^{I}_{\underline{m}},$$

$$E^{\underline{\alpha}} = d\Theta^{\underline{\mu}}V_{\underline{\mu}}{}^{\underline{\alpha}} = (E^{+q}, E^{-}_{\dot{q}}), \qquad E^{+q} = d\Theta^{\underline{\mu}}V_{\underline{\mu}q}{}^{+}, \qquad E^{-}_{\dot{q}} = d\Theta^{\underline{\mu}}V_{\underline{\mu}\dot{q}}{}^{-}. \tag{27}$$

Eq.(27) contains the double–covering of the rotation matrix $U^{\underline{a}}_{\underline{m}}$ (24)

$$V^{\underline{\alpha}}_{\underline{\mu}} = (V^{+}_{\underline{\mu}q},\ V^{-}_{\underline{\mu}\dot{q}}) \quad \in \quad Spin(1,9) \tag{28}$$

It is defined by

$$U^{\underline{a}}_{\underline{m}}\Gamma^{\underline{m}}_{\underline{\mu\nu}} = V^{\underline{\alpha}}_{\underline{\mu}}\Gamma^{\underline{a}}_{\underline{\alpha\epsilon}}V^{\underline{\epsilon}}_{\underline{\nu}}, \qquad U^{\underline{a}}_{\underline{m}}\Gamma^{\underline{\alpha\epsilon}}_{\underline{a}} = V^{\underline{\alpha}}_{\underline{\mu}}\Gamma^{\underline{\mu\nu}}_{\underline{m}}V^{\underline{\epsilon}}_{\underline{\nu}}, \tag{29}$$

and constructed from 8×16 blocks $V^{+}_{\underline{\mu}q}, V^{-}_{\underline{\mu}q}$ which are called spinor Lorentz harmonics [22, 23, 24].

As the space tangent to the Lie group $SO(1,9)$ is isomorphic to the Lie algebra $so(1,9)$ (which is the algebra of $10\times\ 10$ antisymmetric matrices), the derivatives of $U^{a}_{\underline{m}}$ are expressed in terms of antisymmetric *Cartan forms*

$$U^{\underline{a}}_{\underline{m}}dU^{\underline{mb}} = \Omega^{\underline{ab}} = -\Omega^{\underline{ba}} \qquad \Leftrightarrow \qquad dU_{\underline{m}}{}^{\underline{a}} = U_{\underline{m}}{}^{\underline{b}}\Omega_{\underline{b}}{}^{\underline{a}} \tag{30}$$

$$V^{-1}{}_{\underline{\alpha}}{}^{\underline{\mu}}dV_{\underline{\mu}}{}^{\underline{\beta}} = \frac{1}{4}\Omega^{\underline{ab}}(\Gamma_{\underline{ab}})_{\underline{\alpha}}{}^{\underline{\beta}} \qquad \Leftrightarrow \qquad dV_{\underline{\mu}}{}^{\underline{\alpha}} = \frac{1}{4}\Omega^{\underline{ab}}V_{\underline{\mu}}{}^{\underline{\beta}}(\Gamma_{\underline{ab}})_{\underline{\beta}}{}^{\underline{\alpha}} \tag{31}$$

The splittings of U and V into the harmonic components $U^{\pm\pm}_{\underline{m}}, U^{I}_{\underline{m}}$ and $V^{+}_{\underline{\mu}q}, V^{-}_{\underline{\mu}q}$ are invariant under $SO(1,1)\times SO(8)$ local transformations. The Cartan forms $\Omega^{\underline{ab}}$ can be splitted into blocks as well. These blocks are transformed as $SO(1,1)$ spin connections

$$\omega = \frac{1}{2}U^{--}_{\underline{m}}dU^{\underline{m}++}, \tag{32}$$

$SO(8)$ connections (gauge fields)

$$A^{IJ} = U^{I}_{\underline{m}}dU^{\underline{m}J}, \tag{33}$$

and vielbein forms of the coset $\frac{SO(1,9)}{SO(1,1)\times SO(8)}$

$$f^{++I} = U^{++}_{\underline{m}}dU^{\underline{m}I}, \qquad f^{--I} = U^{--}_{\underline{m}}dU^{\underline{m}I}. \tag{34}$$

The Cartan forms (32) -(34) can be used to decompose Eq.(30) as follows

$$\mathcal{D}U^{++}_{\underline{m}} = dU^{++}_{\underline{m}} - U^{++}_{\underline{m}}\omega = f^{++I}U^{I}_{\underline{m}}, \tag{35}$$

$$\mathcal{D}U^I_{\underline{m}} = dU^I_{\underline{m}} + U^I_{\underline{m}}A^{II} = \frac{1}{2}U^{++}_{\underline{m}} f^{--I} + \frac{1}{2}U^{--}_{\underline{m}} f^{++I}. \quad (36)$$

To decompose Eqs. (29), an $SO(1,1)\times SO(8)$ covariant representation of the Mayorana-Weyl 16×16 γ-matrices should be used [24]. In an appropriate representation one obtains

$$\begin{gathered} U^{++}_{\underline{m}}\Gamma^{\underline{m}}_{\underline{\mu\nu}} = 2\ V^{+}_{\underline{\mu}q}V^{+}_{\underline{\nu}q}, \qquad U^{--}_{\underline{m}}\Gamma^{\underline{m}}_{\underline{\mu\nu}} = 2\ V^{-}_{\underline{\mu}q}V^{-}_{\underline{\nu}q}, \qquad U^{I}_{\underline{m}}\Gamma^{\underline{m}}_{\underline{\mu\nu}} = V^{+}_{\underline{\mu}q}\gamma^I_{q\dot q}V^{-}_{\underline{\nu}\dot q} + V^{+}_{\underline{\nu}q}\gamma^I_{q\dot q}V^{-}_{\underline{\mu}\dot q}, \\ \delta_{qp}U^{++}_{\underline{m}} = V^{+}_{q}\tilde\Gamma_{\underline{m}}V^{+}_{p}, \qquad \delta_{\dot q\dot p}U^{--}_{\underline{m}} = V^{-}_{\dot q}\tilde\Gamma_{\underline{m}}V^{-}_{\dot p}, \qquad U^{i}_{\underline{m}}\gamma^i_{q\dot q} = V^{+}_{q}\tilde\Gamma_{\underline{m}}V^{-}_{\dot q}. \end{gathered} \quad (37)$$

Note that Eq. (24) implies

$$U^{\underline{a}}_{\underline{m}}\eta^{\underline{mn}}U^{\underline{b}}_{\underline{n}} = 0 \quad \Leftrightarrow$$

$$U^{++}_{\underline{a}}U^{++\underline{a}} = 0, \qquad U^{--}_{\underline{a}}U^{--\underline{a}} = 0, \quad (38)$$

$$U^{++}_{\underline{a}}U^{--\underline{a}} = 2, \quad (39)$$

$$U^{\pm\pm}_{\underline{a}}U^{i\underline{a}} = 0, \qquad U^{i}_{\underline{a}}U^{j\underline{a}} = -\delta^{ij}. \quad (40)$$

Thus $V^{+}_{\underline{\mu}q}$ and $V^{-}_{\underline{\mu}\dot q}$ can be treated as 'square roots' of the light–like vectors $U^{++}_{\underline{a}}$ and $U^{--}_{\underline{a}}$ (38) normalized by (39).

The integrability conditions for Eqs. (32)–(34) provide us with the *Maurer–Cartan equations* for the Cartan forms

$$\mathcal{D}f^{\pm\pm I} \equiv df^{\pm\pm I} \mp f^{\pm\pm I}\wedge\omega + f^{\pm\pm J}\wedge A^{JI} = 0, \quad (41)$$

$$d\omega = \frac{1}{2}f^{--I}\wedge f^{++I}, \quad (42)$$

$$\mathcal{F}^{IJ} \equiv dA^{IJ} + A^{IK}\wedge A^{KJ} = -f^{--[I}\wedge f^{++J]}. \quad (43)$$

2.2 External derivative of the Lagrangian 2-form and superfield equations of motion

With the above notation it is straightforward to calculate a formal external derivative of the Lagrangian 2-form (22)

$$d\mathcal{L}_2 = \frac{1}{2}\ E^I\wedge(E^{--}\wedge f^{++I} - E^{++}\wedge f^{--I} - 4iE^{+q}\wedge E^{-\dot q}\gamma^I_{q\dot q}) - 2i\ E^{-}_{\dot q}\wedge E^{-}_{\dot q}\wedge E^{++}. \quad (44)$$

The variation of the action (21), (22) can be easily derived from (44) by making use of the seminal formula [9]

[9] Our notation for the external derivative and contraction of a q-form $\Omega_q = \frac{1}{q!}dZ^{\underline{M}_q}\wedge\ldots\wedge dZ^{\underline{M}_1}\Omega_{\underline{M}_1\ldots\underline{M}_q}(Z)$ is as follows

$$d\Omega_q = \frac{1}{q!}dZ^{\underline{M}_q}\wedge\ldots\wedge dZ^{\underline{M}_1}\wedge dZ^{\underline{N}}\partial_{\underline{N}}\Omega_{\underline{M}_1\ldots\underline{M}_q}(Z), \qquad i_\delta\Omega_q = \frac{1}{(q-1)!}dZ^{\underline{M}_q}\wedge\ldots\wedge dZ^{\underline{M}_2}\delta Z^{\underline{M}_1}\Omega_{\underline{M}_1,\underline{M}_2\ldots\underline{M}_q}(Z)$$

$$\delta \mathcal{L}_2 = i_\delta d\mathcal{L}_2 + d i_\delta \mathcal{L}_2 \tag{45}$$

supplemented by the rules

$$i_\delta dZ^{\underline{M}} = \delta \hat{Z}^{\underline{M}}. \tag{46}$$

The contractions of Cartan forms $i_\delta f^{++I}$, $i_\delta f^{--I}$ should be considered as parameters of independent variations of the Lorentz harmonic superfields U and V.

The equations of motion which follow from generalized action are

$$\hat{E}^I = \hat{\Pi}^{\underline{m}} U^I_{\underline{m}} = 0, \tag{47}$$

$$\hat{\Psi}_{2\dot{q}}^{+} \equiv E^{++} \wedge E^{-}_{\dot{q}} = 0, \tag{48}$$

$$\hat{M}_2^I = \frac{1}{2} E^{++} \wedge f^{--I} - \frac{1}{2} E^{--} \wedge f^{++I} - 2i E^{+}_{q} \wedge E^{-}_{\dot{q}} \gamma^I_{q\dot{q}} = 0, \tag{49}$$

The key point is that Eqs.(47)-(49) can be regarded as superfield equations which *are valid in the whole worldvolume superspace* [29]. In such a treatment Eq. (47) (the basic superembedding equation) is an equivalent form of geometrodynamic condition (20) [25]. It does not contain the fermionic equations of motion (48) among its consequences. Studying the integrability conditions for Eq.(47)

$$\mathcal{D}\hat{E}^I = d\hat{E}^I + A^{IJ} \wedge \hat{E}^J = \frac{1}{2} \hat{E}^{--} \wedge f^{++I} + \frac{1}{2} \hat{E}^{++} \wedge f^{--I} - 2i \hat{E}^{+}_{q} \wedge \hat{E}^{-}_{\dot{q}} \gamma^I_{q\dot{q}} = 0, \tag{50}$$

one finds that (see Section 6)

$$\hat{E}^{-}_{\dot{q}} = \hat{E}^{++} \psi_{++\dot{q}}^{\ \ -} + \hat{E}^{--} \psi_{--\dot{q}}^{\ \ -}, \tag{51}$$

while, as it is easy to see, Eq.(48) also implies $\psi^{-}_{--\dot{q}} = 0$. The latter is just the fermionic equation of motion. Indeed, in the linear approximation it reduces to $\psi^{-}_{--\dot{q}} \approx \partial_{--} \Theta^{-}_{\dot{q}} = 0$, where $\Theta^{-}_{\dot{q}} = \Theta^{\underline{\mu}} V^{-}_{\underline{\mu}\dot{q}}$.

On the other hand, one finds that the superembedding equation (47) and fermionic equations of motion (48) determine the bosonic equations (48) completely. Indeed, on the surface of the superembedding equation (47), where Eq. (50) holds, the bosonic equation of motion (48) can be written in the form

$$\hat{M}_2^I|_{\hat{E}^I=0} = E^{++} \wedge f^{--I} - 4i E^{+}_{q} \wedge E^{-}_{\dot{q}} \gamma^I_{q\dot{q}} = 0. \tag{52}$$

Then one easily finds that Eq. (52) can be obtained as a derivative of the fermionic equation (48)

$$\mathcal{D}(\Psi_2)^{-}_{\dot{q}}\ |_{\hat{E}^I=0} = -\frac{1}{2} \hat{M}_2^I \wedge E^{+}_{q} \gamma^I_{q\dot{q}}. \tag{53}$$

An important property of the Lagrangian 2−form Eq.(22) is that it is closed on the surface of the superembedding equation Eq.(47)

$$d\hat{\mathcal{L}}_2|_{\hat{E}^I=0} \equiv 0 \tag{54}$$

Indeed, the pull–back of the first term in Eq. (44) vanishes due to the superembedding equation (47) while the second one becomes zero when the consequence (51) of Eq.(47) is taken into account. As Eq.(47) is an equivalent form of the geometrodynamic condition (20), one can say that the Lagrangian form (22) of the generalized action (21) possesses the property called Weyl triviality [10].

Now we are ready to proceed with the superfield action.

3 New version of superfield action for $D = 10$, $N = 1$ closed superstring

We propose the following superfield action functional for the $D = 10$, $N = 1$ closed superstring

$$S = \int d^2\xi \hat{d}^8\eta \left(P_I^M \hat{E}_M^I + P^{MN}(\hat{\mathcal{L}}_2 - dY_1)_{NM} \right). \tag{55}$$

Here the superfield $\hat{E}_M^I$ emerges in the decomposition of the pull-back of the bosonic vielbein form $\hat{E}^I = \hat{\Pi}^{\underline{m}} U_{\underline{m}}^I$ on the worldsheet superspace $\sum^{(2|8)} = (\xi^m, \eta^q) = (\zeta^M)$

$$\hat{E}^I = d\zeta^M \hat{E}_M^I = \hat{\Pi}^{\underline{m}} U_{\underline{m}}^I, \qquad \Pi^{\underline{m}} = dX^{\underline{m}} - id\Theta\Gamma^{\underline{m}}\Theta, \tag{56}$$

$\hat{\mathcal{L}}_2$ is the Lagrangian form of the generalized action (22), but pulled-back onto the whole worldsheet superspace instead of a bosonic surface in this superspace

$$\hat{\mathcal{L}}_2 = \frac{1}{2}\hat{E}^{++} \wedge \hat{E}^{--} - i\hat{\Pi}^{\underline{m}} \wedge d\hat{\Theta}\Gamma_{\underline{m}}\hat{\Theta} = \frac{1}{2} d\zeta^M \wedge d\zeta^N \hat{\mathcal{L}}_{NM}. \tag{57}$$

Thus

$$(\hat{\mathcal{L}}_2)_{NM} = \hat{E}_{[N}^{--} \hat{E}_{M)}^{++} - 2\, i\partial_{[N|}\hat{\Theta}\Gamma_{\underline{m}}\hat{\Theta}\hat{\Pi}_{|\underline{M})}^{\underline{m}} \tag{58}$$

where mixed brackets $[\ldots)$ denote graded antisymmetrization with common weight unity, e.g.

$$E_{[N}^{--} E_{M)}^{++} \equiv \frac{1}{2}(E_N^{--} E_M^{++} - (-)^{MN} E_M^{--} E_N^{++}). \tag{59}$$

$P^{MN} = P^{[MN)}(\zeta)$ and $P_I^M(\zeta)$ are Lagrange multiplier superfields and

$$Y_1 \equiv d\zeta^M Y_M(\zeta) \tag{60}$$

is an auxiliary 1−form superfield (cf. [12]). (Thus $(dY_1) = d\zeta^M \wedge d\zeta^N \partial_N Y_M$, $\quad (dY_1)_{NM} = 2\partial_{[N} Y_{M]} = \partial_N Y_M - (-)^{NM} \partial_M Y_N$).

3.1 Variation of the action

Usually the variation of the actions with superspace tensors is quite involved due to many sign factors which appear in calculations. However, for the action Eq.(55) there exists a shortcut which is provided by the fact that all the expressions involved (except for the Lagrange multipliers) are expressed in terms of differential forms. Hence, to vary different terms of the action one can use Eq. (45) and, then, extract the basic differential forms $d\zeta^M$, $d\zeta^M \wedge d\zeta^N$. For instance, using the expression (50) for the external derivative of the form $\hat{E}^I = \Pi^{\underline{m}} U_{\underline{m}}^I$ and Eq. (45) one easily arrives at

$$\delta E_M^I = \partial_M (i_\delta \Pi^{\underline{m}}) U_{\underline{m}}^I - E_M^J i_\delta A^{JI} + \frac{1}{2} E_M^{--} i_\delta f^{++I} + \frac{1}{2} E_M^{++} i_\delta f^{--I} - \tag{61}$$

$$-2iE_{Mq}^{+} \gamma_{q\dot{q}}^I i_\delta E_{\dot{q}}^{-} - 2iE_{M\dot{q}}^{-} \gamma_{q\dot{q}}^I i_\delta E_q^{+}.$$

Here and below we will skip the hat symbol $\hat{\ldots}$ from the pull–backs of differential forms for the sake of shortness (i.e. $\hat{E}^a$ reads now as E^a).

The variation of $(\mathcal{L}_2)_{NM}$ can be found in the similar way from Eqs.(44) and (45). Making use of this technique we arrive at the following expression for the variation of superfield action (55)

$$\delta S = \int d^2\xi \hat{d}^8\eta \Big[(\delta P_I^M + P_J^M i_\delta A^{JI}) E_M^I + \delta P^{MN} (\mathcal{L}_2 - dY_1)_{NM} - \tag{62}$$

$$-(-)^N \partial_N P^{NM} (\delta Y_M - (i_\delta \mathcal{L}_2)_M) +$$

$$+(\frac{1}{2} P_I^M E_M^{++} - P^{MN} E_N^{++} E_M^I) i_\delta f^{--I} + (\frac{1}{2} P_I^M E_M^{--} + P^{MN} E_N^{--} E_M^I) i_\delta f^{++I} +$$

$$+ \left(-(-)^M (\partial_M P_I^M - A_M^{IJ} P_J^M) + P^{MN} (f_N^{++I} E_M^{--} - f_N^{--I} E_M^{++} + 4i E_{N\dot{q}}^{-} E_{Mq}^{+} (-)^M \gamma_{q\dot{q}}^I \right) i_\delta E^I -$$

$$- \left(\frac{1}{2} P_I^M f_M^{++I} + P^{MN} f_N^{++I} E_M^I \right) i_\delta E^{--} -$$

$$- \frac{1}{2} \left(P_I^M f_M^{--I} + 8i P^{MN} \left(-(-)^M E_{N\dot{q}}^{-} E_{M\dot{q}}^{-} + \frac{i}{4} f_N^{--I} E_M^I \right) \right) i_\delta E^{++} -$$

$$-2i \left(P_I^M E_{Mq}^{+} \gamma_{q\dot{q}}^I - 4 P^{MN} \left(E_{N\dot{q}}^{-} E_M^{++} - \frac{1}{2} E_{Nq}^{+} \gamma_{q\dot{q}}^I E_M^I \right) \right) i_\delta E_{\dot{q}}{}^{-} -$$

$$-2i \left(P_I^M E_{M\dot{q}}^{-} \gamma_{q\dot{q}}^I - 2 P^{MN} E_{N\dot{q}}^{-} \gamma_{q\dot{q}}^I (-)^M E_M^I \right) i_\delta E_q{}^{+} \Big].$$

Let us turn to the equations of motion. The variations of the Lagrange multipliers P_I^M, P^{MN} produce the superembedding equation (47)

$$\delta P_I^M : \qquad E_M^I = 0 \quad \Rightarrow \quad E^I \equiv \Pi^{\underline{m}} U_{\underline{m}}^I = 0 \tag{63}$$

and the equation

$$\delta P^{MN} : \qquad (\mathcal{L}_2 - dY_1)_{NM} = 0 \quad \Rightarrow \quad \mathcal{L}_2 = dY_1. \tag{64}$$

Since the integrability conditions for Eq.(64)

$$d\mathcal{L}_2 = 0 \tag{65}$$

are satisfied identically on the surface of the superembedding equation (63) (see Eq.(54)), Eq.(64) expresses the auxiliary super–1–form Y_1 (60) through $\mathcal{L}_2$ and thus is not a dynamical equation. To arrive at such a conclusion one should recall that $\delta Y_M = \partial_M f$ is a gauge symmetry of the action (see Eq.(62)).

On the other hand, the above mentioned dependence of the integrability conditions (65) for Eq.(64) is nothing but the Noether identity which reflects the presence of additional gauge symmetry with the basic relation $\delta P^{MN} \equiv (-)^K \partial_K \Sigma^{KMN}$, $\Sigma^{KMN} = \Sigma^{[KMN)}$. The Noether identity for the $SO(8)$ symmetry is manifested by the fact that the equations of motion which emerge as a result of the variation $i_\delta A^{IJ}$:

$$i_\delta A^{IJ} : \qquad P_I^M E_M^J - P_J^M E_M^I = 0 \tag{66}$$

are satisfied identically when Eq.(63) is taken into account. We turn to further study of Noether identities and gauge symmetries in Section 6.

The variation with respect to the auxiliary 2-form superfield $\delta Y_M(\zeta)$ provides the equation

$$(-)^N \partial_N P^{NM} = 0. \tag{67}$$

The remaining equations of motion on the surface of the superembedding condition (63) have the form

$$i_\delta f^{--I}: \qquad P_I^M E_M^{++} = 0, \tag{68}$$

$$i_\delta f^{++I}: \qquad P_I^M E_M^{--} = 0, \tag{69}$$

$$i_\delta E^I: \qquad (-)^M (\partial_M P_I^M - A_M^{IJ} P_J^M) = \tag{70}$$

$$= P^{MN} (f_N^{++I} E_M^{--} - f_N^{--I} E_M^{++} + 4i(-)^M E_{N\dot{q}}^{-} E_{Mq}^{+} \gamma_{q\dot{q}}^I),$$

$$i_\delta E^{--}: \qquad P_I^M f_M^{++I} = 0, \tag{71}$$

$$i_\delta E^{++}: \qquad P_I^M f_M^{--I} = +4i P^{MN} (-)^M E_{N\dot{q}}^{-} E_{M\dot{q}}^{-}, \tag{72}$$

$$i_\delta E_{\dot{q}}^{-}: \qquad P_I^M \gamma_{q\dot{q}}^I E_{Mq}^{+} = 4 P^{MN} (-)^M E_{N\dot{q}}^{-} E_M^{++}, \tag{73}$$

$$i_\delta E_q^{+}: \qquad P_I^M E_{M\dot{q}}^{-} \gamma_{q\dot{q}}^I = 0. \tag{74}$$

Eqs.(67), (70) look like dynamical ones. However, as we will see in Section 6, there exist gauge symmetries which make possible to gauge away the general solution of these equations. That means that the Lagrange multiplier superfields do not contain dynamical degrees of freedom. Due to this fact the model described by the action Eq.(55) is equivalent to the $D = 10$, $N = 1$ Green - Schwarz superstring at the classical level.

4 Generation of supergravity constraints

The action (55), (56), (57) does not contain an intrinsic worldsheet supervielbein

$$e^A = (e^{++}, e^{--}, e^{+q}) = d\zeta^M e_M^A \tag{75}$$

and intrinsic spin connections as independent variables. In this sense Eq.(55) can be regarded as a superfield counterpart of Nambu-Goto action, while the original superfield action [12] can be treated as a counterpart of the Brink–Di Vecchia–Howe–Polyakov action.

However, to deal with the action, in particular to investigate equations of motion and to study symmetries, one needs to assume that among the pull–backs of the target space supervielbein there exists a set of two bosonic and 8 fermionic forms which are linearly independent. We assume that these forms are

$$\hat{E}^{\pm\pm} = \hat{\Pi}^{\underline{m}} U_{\underline{m}}^{\pm\pm} = d\zeta^M \hat{E}_M^{\pm\pm}(\zeta), \qquad \hat{E}_M^{+q} = d\zeta^M \hat{E}_M^{+q}(\zeta) \tag{76}$$

Thus the inverse blocks

$$\hat{E}_{\pm\pm}^{\ M}, \hat{E}_{+q}^{\ M}, \tag{77}$$

do exist and can be defined by

$$\hat{E}_{+q}^M \hat{E}_M^{+p} = \delta_q^p, \qquad \hat{E}_{+q}^M \ \hat{E}_M^{\pm\pm} \ = 0, \tag{78}$$

$$\hat{E}_{\pm\pm}^M \hat{E}_M^{+q} \ = 0, \qquad \hat{E}_{+q}^M \ \hat{E}_M^{\pm\pm} = 0, \qquad \hat{E}_{++}^M \ \hat{E}_M^{--} = 0 = \hat{E}_{--}^M \ \hat{E}_M^{++}.$$

Actually, by this step we implicitly assumed that the worldsheet supervielbein can be induced by embedding in accordance with the relations

$$e^{\pm\pm} = \hat{E}^{\pm\pm} \equiv \hat{\Pi}^{\underline{m}} U_{\underline{m}}^{\pm\pm}, \qquad e^{+q} = \hat{E}_M^{+q} \equiv d\hat{\Theta}^{\underline{\mu}} V_{\underline{\mu}q}^{\ \ +}. \tag{79}$$

To complete the description of the worldsheet geometry one can choose the worldsheet spin connection and the $SO(8)$ gauge field to be equal to the Cartan forms (32), (33)

$$\omega = \frac{1}{2} U_{\underline{m}}^{--} dU^{\underline{m}++}, \qquad A^{IJ} = U_{\underline{m}}^{I} dU^{\underline{m}J}. \tag{80}$$

Then the worldsheet torsions are defined by

$$T^{++} = \mathcal{D}e^{++} = de^{++} - e^{++} \wedge \omega, \tag{81}$$

$$T^{--} = \mathcal{D}e^{--} = de^{--} + e^{--} \wedge \omega, \tag{82}$$

$$T^{+q} = \mathcal{D}e^{+q} = de^{+q} - \frac{1}{2} e^{+q} \wedge \omega + \frac{1}{4} e^{+p} \wedge A^{IJ} \gamma^I_{p\dot{p}} \gamma^J_{q\dot{p}} \tag{83}$$

and *can be calculated directly* from Eqs. (79) with the use of Eqs. (23), (27), (35), (36) and the superembedding equation (63).

To proceed in this way, one shall use the general decomposition for the remaining fermionic supervielbein forms

$$\hat{E}^-_{\dot{q}} = e^{+q} a^{--}_{q\dot{q}} + e^{++} \psi^-_{++\dot{q}} + e^{--} \psi^-_{--\dot{q}}, \tag{84}$$

as well as for the covariant Cartan forms (34), e.g.

$$f^{++I} = e^{+q} f^{++I}_{+q} + e^{++} f^{++I}_{++} + e^{--} f^{++I}_{--}. \tag{85}$$

As a result of calculations one arrives at the standard constraints for the bosonic torsion (81)

$$T^{++} = 2i e^{+q} \wedge e^{+q}. \tag{86}$$

The expressions for the torsion forms (82), (83) are more complicated. On the surface of the superembedding condition (63) they can be written as

$$T^{--} = -2i E^-_{\dot{q}} \wedge E^-_{\dot{q}}, \tag{87}$$

$$T^{+q} = \frac{1}{2} E^-_{\dot{q}} \wedge f^{++I} \gamma^I_{q\dot{q}}, \tag{88}$$

where $E^-_{\dot{q}}$ and f^{++I} are defined in Eqs. (84), (85). The curvature and gauge field strength are determined from the Maurer–Cartan equations (42), (43)

$$d\omega = \frac{1}{2} f^{--I} \wedge f^{++I}, \tag{89}$$

$$\mathcal{F}^{IJ} \equiv dA^{IJ} + A^{IK} \wedge A^{KJ} = -f^{--[I} \wedge f^{++J]}.$$

The constraints are essentially simplified when all the superfield equations of motion are taken into account (see below).

5 Investigation of superfield equations

It is convenient to begin the complete investigation of the superfield equations (63), (67)–(74) with the study of the integrability conditions (50) of the superembedding equation (63). To this end we assume that the worldsheet geometry induced by embedding in accordance with Eqs. (79), (80) and substitute the most general expression (84) for remaining fermionic supervielbein forms. The result can be decomposed into the basic 2–forms $e^{+q} \wedge e^{+p}$, $e^{\pm\pm} \wedge e^{+p}$, $e^{++} \wedge e^{--}$. The coefficients $(\mathcal{D}E^I)_{+q+p}$, $(\mathcal{D}E^I)_{+q\pm\pm}$, $(\mathcal{D}E^I)_{--\ ++}$, which emerge in the product with basic 2–forms, are superfield equations. It is convenient to classify them by the dimensionality of the basic forms $[e^{+q} \wedge e^{+p}] = 1$, $[e^{\pm\pm} \wedge e^{+p}] = 3/2$, $[e^{++} \wedge e^{--}] = 2$.

At dimension 1 one gets

$$(\frac{i}{4}\mathcal{D}E^I)_{+q+p} = a_{(p\dot{q}}\gamma^I_{q)\dot{q}} = 0. \tag{90}$$

Substituting the most general decomposition $a_{q\dot{q}} = a^I\gamma^I_{q\dot{q}} + a^{J_1J_2J_3}\gamma^{J_1J_2J_3}_{q\dot{q}}$ one can obtain an equivalent form of Eq. (90)

$$a^I\delta_{qp} + a^{J_1J_2J_3}\gamma^{IJ_1J_2J_3}_{qp} = 0, \tag{91}$$

which evidently implies $a^I = 0$, $a^{J_1J_2J_3} = 0$. Hence the general solution of Eq. (90) is trivial

$$a_{q\dot{q}} = 0. \tag{92}$$

Thus a consequence of superembedding equation (63) is that the fermionic supervielbein 1–form $E^-_{\dot{q}}$ has the form (51), or, equivalently

$$\hat{E}^-_{\dot{q}} = e^{++}\psi_{++\dot{q}}^{\ -} + e^{--}\psi_{--\dot{q}}^{\ -}. \tag{93}$$

The equations of dimensions 3/2 and 2 provide us with the expressions for the covariant Cartan forms (34)

$$f^{++I} = -4ie^{+q}\gamma^I_{q\dot{q}}\psi_{--\dot{q}}^{\ -} + e^{++}h^I + e^{--}f^{++I}_{--}, \tag{94}$$

$$f^{--I} = -4ie^{+q}\gamma^I_{q\dot{q}}\psi_{++\dot{q}}^{\ -} + e^{++}f^{--I}_{++} + e^{--}h^I. \tag{95}$$

(Actually the simplest way to derive (94), (95) is to substitute (93) back into Eq. (50)).

It is instructive to find the expressions for the left–hand–sides of bosonic and fermionic superfield equations (49), (48), which we obtained in the frame of generalized action principle. They are

$$\hat{M}^I_2 = e^{++} \wedge e^{--}h^I + 4ie^{--} \wedge e^{+q}\psi_{--\dot{q}}^{\ -}\gamma^I_{q\dot{q}}, \tag{96}$$

$$\hat{\Psi}_{2\dot{q}}^{\ -} = e^{++} \wedge e^{--}\psi_{--\dot{q}}^{\ -}. \tag{97}$$

The leading component $h^I_0 = h^I|_{\eta=0}$ of the superfield h^I is known as the mean curvature of the worldsheet. The bosonic equations is essentially $h^I_0 = 0$ and the fermionic ones are $(\psi_{--\dot{q}}^{\ -})_0 \equiv \psi_{--\dot{q}}^{\ -}|_{\eta=0} = 0$. However, by now we have not obtained them from the action.

Substituting (93), (94), (95) into Eqs. (87), (88) one arrives at the torsion constraints for induced worldsheet supergravity on the surface of superembedding equation (63)

$$T^{++} = 2ie^{+q} \wedge e^{+q}, \tag{98}$$

$$T^{--} = -4ie^{++} \wedge e^{--}\psi_{++\dot{q}}^{\;-}\psi_{--\dot{q}}^{\;-}, \tag{99}$$

$$T^{+q} = 2ie^{\pm\pm} \wedge e^{+q}\gamma^I_{q\dot{q}}\psi_{\pm\pm\dot{q}}^{\;-} + e^{++} \wedge e^{--}\left(f_{--}^{++I}\gamma^I_{q\dot{q}}\psi_{++\dot{q}}^{\;-} - h^I\gamma^I_{q\dot{q}}\psi_{--\dot{q}}^{\;-}\right). \tag{100}$$

After making substitution of Eq. (94), the lowest dimensional component of the Peterson-Codazzi equation (41) results in

$$\gamma^I_{(q\dot{q}}\mathcal{D}_{+p)}\psi_{--\dot{q}}^{\;-} = -\frac{1}{2}\delta_{qp}h^I \tag{101}$$

Eq. (101) implies the dependence of the bosonic superfield equation of motion which follows from the generalized action on the fermionic equation. It can be 'solved' algebraically by (cf. (53))

$$\mathcal{D}_{+p}\psi_{--\dot{q}}^{\;-} = -\frac{1}{2}\gamma^I_{q\dot{q}}h^I. \tag{102}$$

6 Noether identities and hidden gauge symmetries

To carry out the analysis of dynamical degrees of freedom one needs to know all the gauge symmetries. From the standard approach [11, 12] it is known that there should be some hidden gauge symmetries acting on the Lagrange multiplier superfields which reduce their dynamical content. However, the analysis of gauge symmetries inherent to a superfield action is rather involved.

To overcome the cumbersome calculations we propose to use the second Noether theorem. It states that any gauge symmetry of the action results in a dependence of equations of motion, and that the converse is true as well, i.e. *an interdependence of equations of motion indicates the presence of a gauge symmetry*.

In our case the second Noether theorem allows to state that the superfield action (55) possesses the gauge symmetries with basic terms in the transformation low defined by

$$\delta P^{NM} = (-)^K \partial_K \Sigma^{KMN}, \tag{103}$$

$$\delta P_I^M = 2(-)^N \partial_N S^{+q+pI} E_{+q}^{\;N} E_{+q}^{\;M} (-)^N + \ldots \tag{104}$$

where $\ldots$ denotes the terms dependent either on the superfield parameter $\Sigma^{KMN} = -(-)^{MN}\Sigma^{KNM} = \Sigma^{[KMN)}$ (graded–antisymmetric supertensor) or on the parameter S^{+q+pI} which obey the properties [10]

$$\tilde{S}^{+q+pI} = \tilde{S}^{+p+qI}, \qquad \tilde{S}^{+p+qI}\gamma^I_{q\dot{q}} = 0, \quad \Rightarrow \quad \tilde{S}^{+q+qI} = 0. \tag{105}$$

The dependence (54) of the integrability conditions (65) for the equation (64)

$$E^I = 0, \quad (\mathcal{D}E^I)_{+q+p} = 0, \qquad \Rightarrow \qquad d\mathcal{L}_2 = 0, \tag{106}$$

[10]The terms dependent on S^{+q+pI} in (104) involve it in products with different superfields and are unessential for the analysis of dynamical degrees of freedom as the latter can be carried out in linear approximation.

mentioned in Section 3, indicates the presence of the symmetry with basic transformation low (103). Indeed, if one considers the variation of the action (55) under the Lagrange multiplier transformations (103), one easily finds (cf. (62))

$$\delta_0 S = -1/3 \int d^2\xi d^2\eta \Sigma^{MNK} (d\mathcal{L}_2)_{KNM} \tag{107}$$

But $d\mathcal{L}_2$ vanishes as a result of Eq. (63), which emerges as a result of the variation of the Lagrange multiplier P_I^M, and of its integrability condition (106) only. This guarantees that **i)** one can find transformations for the Lagrange multiplier superfield P_I^M which will compensate the variation (107), and that **ii)** other superfields can be regarded to be inert under the gauge symmetry (103).

To establish the presence of the gauge symmetry (104) one can turn back to the lowest dimensional component (90) of the integrability condition (50) for Eq. (63). Eq. (90) carries a $\mathbf{36}_s \times \mathbf{8}_v = \mathbf{288}$ reducible representation of the $SO(8)$ group. Its decomposition onto irreducible representations reads

$$\mathbf{288} = \mathbf{8} + \mathbf{56} + \mathbf{224}, \qquad \Leftrightarrow \qquad \begin{smallmatrix}\square\\ \square\end{smallmatrix} \otimes \square = \square + \square\square\square + \begin{smallmatrix}\square\square\square\square\\ \square\end{smallmatrix} \tag{108}$$

or more explicitly

$$(\mathcal{D}E^I)_{+q+p} = s^I \delta_{qp} + s^{I_1I_2I_3}\gamma_{qp}^{II_1I_2I_3} + s^{I,I_1I_2I_3I_4}\gamma_{qp}^{I_1I_2I_3I_4} \tag{109}$$

where

$$s^{I,J_1J_2J_3J_4} = s^{J_1,IJ_2J_3J_4} = \frac{1}{4!}\varepsilon^{J_1J_2J_3J_4I_1I_2I_3I_4} s^{I,I_1I_2I_3I_4} \equiv \begin{smallmatrix}\square\square\square\square\\ \square\end{smallmatrix} \tag{110}$$

is the $SO(8)$ tensor description of the **224** irreducible representation.

One can see that to arrive at the trivial solution (92) it is enough to consider only **8** and **56** irreducible parts of Eq.(90):

$$s^I =\propto (\mathcal{D}E^I)_{+q+q} = 0, \quad s^{J_1J_2J_3} =\propto (\mathcal{D}E^I)_{+q+p}\gamma_{qp}^{IJ_1J_2J_3} = 0 \qquad \Rightarrow \qquad a_{q\dot{q}} = 0. \tag{111}$$

Thus the **224** irreducible part of Eq. (90) is satisfied identically due to **8** and **56** irreducible parts of this equation. This is the Noether identity for the gauge symmetry (104) with

$$S^{+q+pI} = S^{I,I_1I_2I_3I_4}\gamma_{qp}^{I_1I_2I_3I_4}, \tag{112}$$

$$S^{I,J_1J_2J_3J_4} = S^{J_1,IJ_2J_3J_4} = \frac{1}{4!}\varepsilon^{J_1J_2J_3J_4I_1I_2I_3I_4} S^{I,I_1I_2I_3I_4} \equiv \mathbf{224} \equiv \begin{smallmatrix}\square\square\square\square\\ \square\end{smallmatrix}, \tag{113}$$

Eq. (112) provides the general solution of the gamma–tracelessness conditions for the vector–spin-tensor S^{+q+pI} (105).

Hence, using the second Noether theorem we have proved that the superfield action (55) possesses hidden gauge symmetries (103), (104).

7 The fate of Lagrange multiplier superfields and component equations of motion

The gauge symmetry (103) and the equation of motion (67) imply that the Lagrange multiplier superfield P^{MN} do not carry dynamical degrees of freedom. Indeed, the general solution of Eq.(67) has the form [12] ($(\eta)^8 \equiv 1/8!\epsilon_{q_1\ldots q_8}\eta^{q_1}\ldots\eta^{q_8}$ (5))

$$P^{NM} = (-)^K \partial_K \tilde{\Sigma}^{KNM} + \delta_n^N \delta_m^M \epsilon^{mn} (\eta)^8 \, T, \qquad dT = 0. \tag{114}$$

The first term in Eq.(114) can be gauged away by the transformations Eqs.(103), (104) with parameter $\Sigma^{KNM} = -\tilde{\Sigma}^{KNM}$. In this gauge one gets

$$P^{NM} = \delta_n^N \delta_m^M \epsilon^{mn} (\eta)^8 \, T, \qquad dT = 0 \tag{115}$$

and finds that the constant T is the string tension.

The situation with the Lagrange multiplier P_I^M is slightly more complicated. The general solution of Eqs.(68), (69) is

$$P_I^M = P_I^{+q} E_{+q}^{\ M}. \tag{116}$$

At this point it is pertinent to note that Eq. (74) is satisfied identically when Eqs. (116) and (93) are taken into account. The identification (79) makes evident that this dependence reflects $n = (8, 0)$ *local worldsheet supersymmetry* on the language of Noether identities. The bosonic reparamterization is reflected by the fact that Eqs. (71), (72) are dependent on Eqs. (93), (116) and (73).

In the gauge (115) the Eq. (73) acquires the form

$$P_I^{+q} \gamma^I_{q\dot{q}} = -4T(\eta)^8 e(\zeta) \psi_{--\dot{q}}^{\ -} \tag{117}$$

where

$$e(\zeta) = e(\xi, \eta) = \frac{1}{2} \epsilon^{mn} e_m^{++} e_n^{--}, \tag{118}$$

and Eq. (70) becomes

$$(-)^M (\partial_M (P_I^{+q} E_{+q}^M) + A_{+q}^{IJ} P_J^{+q}) = 2(\eta)^8 \, T \, e \, \tilde{h}^I, \tag{119}$$

where

$$\tilde{h}^I = h^I + \frac{2i}{e} \epsilon^{mn} e_m^{--} e_n^{+q} \psi_{--\dot{q}}^{\ -} \tag{120}$$

Eqs. (119), (117) and the gauge symmetry (104) with the parameter (105) imply that
i) the Lagrange multiplier superfield P_I^M does not contain independent dynamical degrees of freedom,
ii) the component coordinate equations of motion for superstring

$$(\psi_{--\dot{q}}^{\ -})_0 = 0, \tag{121}$$

$$(h^I)_0 = 0 \tag{122}$$

are satisfied.

We prove this fact in the Appendix. Similar mechanism was discovered for the first time in Ref. [14] where superfield actions for supermembrane in D=4,5,7 was constructed and an STV-like action for a 2–dimensional extended object in D=11 was considered.

Thus we conclude that the superfield action Eq.(55) indeed describes the $D = 10$, $N = 1$ closed Green-Schwarz superstring.

Conclusion

In this paper we constructed a new version of superfield action for the $D = 10$, $N = 1$ closed superstring (i.e. a heterotic superstring without heterotic fermions). The action possesses manifest $n = (8, 0)$ worldsheet supersymmetry and involves Lorentz harmonics as auxiliary superfields. The second ('Wess-Zumino' [12]) term of the action is constructed from the Lagrangian form of the generalized action principle [29].

To find the hidden gauge symmetries which allow one to remove redundant degrees of freedom from the Lagrange Multiplier superfields we used the second Noether theorem.

In distinction to the original action [12] and to the action from [16] our functional does not contain worldsheet supervielbein e_M^A explicitly. Thus our action can be considered as a superfield counterpart of the Nambu–Goto action while the original one [12] should be regarded as a counterpart of the Brink–Di Vecchia–Howe–Polyakov functional. This property is important because the worldsheet supervielbein should be either constructed from 'prepotentials' or subject to a set of torsion constraints. This is not a problem for the $N = 1$ superstring, but could produce some difficulties for the construction of superfield actions for other branes with high dimensional worldsheet $p > 1$ and $n \leq 8$ worldsheet supersymmetries. (E.g. the off–shell description of $d = 4$, $n = 2$ superfield supergravity is impossible without the use of harmonic variables [35]).

To deal with the new version of superfield action (55) we do not need any assumptions about worldsheet supergravity. Instead we have to assume that some components (76) of the pull–back of the supervielbein of the target superspace form an invertible 10×10 supermatrix (cf. (78))

$$\hat{E}_M^{\ A} = (\hat{E}_M^{++}, \hat{E}_M^{--}, \hat{E}_M^{+q}), \qquad sdet(\hat{E}_M^{\ A}) \neq 0. \tag{123}$$

In such a way we actually perform (implicitly) an identification of the form $\hat{E}^A = d\zeta^M E_M^A \left(\hat{Z}(\zeta)\right)$ with a worldsheet supervielbein $e^A = (e^{++}, e^{--}, e^{+q})$. The worldsheet spin connection ω_M and $SO(8)$ gauge fields A_M^{IJ} can be constructed from Lorentz harmonic superfields (and target superspace spin connections when the general $D = 10$, $N = 1$ supergravity background is considered).

Thus the geometry of the worldsheet superspace is induced by embedding in accordance with the following rules

$$e^{\pm\pm} = \hat{E}^{\pm\pm} \equiv \hat{\Pi}^{\underline{m}} U_{\underline{m}}^{\pm\pm}, \qquad e^{+q} = \hat{E}_M^{+q} \equiv d\hat{\Theta}^{\underline{\mu}} V_{\underline{\mu}q}^{\ \ +}, \tag{124}$$

$$\omega = \frac{1}{2} U_{\underline{m}}^{--} dU^{\underline{m}++}, \qquad A^{IJ} = U_{\underline{m}}^{I} dU^{\underline{m}J}.$$

The worldsheet supergravity constraints can be obtained as integrability conditions for Eqs. (124) taken on the surface of superembedding equations and have the form (98), (99), (100). Such a mechanism of the generation of worldsheet supergravity constraints is characteristic of the generalized action [29]. Its bosonic counterpart was used in [39] in an early consideration of the Brane–World scenario.

Note that the new formulation provides as well a 'complete twistorization' of $N = 1$ (heterotic) superstring in a way alternative to the one proposed in Ref. [16]. Indeed, the basic superembedding equation (63) and Eq. (124) imply that

$$\Pi^{\underline{m}} = \frac{1}{2} e^{++} U^{\underline{m}--} + \frac{1}{2} e^{--} U^{\underline{m}++}$$

Thus we arrive at

$$\Pi^{\underline{m}}_{++} = \frac{1}{2} U^{\underline{m}--} \equiv \frac{1}{16} V^{-}_{\dot{q}} \tilde{\Gamma}_{\underline{m}} V^{-}_{\dot{p}}, \qquad \Pi^{\underline{m}}_{--} = \frac{1}{2} U^{\underline{m}++} \equiv \frac{1}{16} V^{+}_{q} \tilde{\Gamma}_{\underline{m}} V^{+}_{p}.$$

These relations provide a twistor–like solution for the Virasoro constraints (cf. (38), (37)) $\Pi^{\underline{m}}_{++}\Pi_{++\underline{m}} = 0$, $\Pi^{\underline{m}}_{++}\Pi_{++\underline{m}} = 0$.

The methods developed in this paper should simplify the investigation of superfield actions for higher superbranes in low dimensions and can be useful for studying the physical contents of 'spinning superbrane' models in $D = 11$ and $D = 10$ type II superspaces which are described by STV-like actions with 16 worldvolume supersymmetries [14, 15].

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Appendix

Here we present some details of a direct derivation of the gauge symmetry (104) and use it to prove that
i) the Lagrange multiplier superfield P_I^M does not contain independent dynamical degrees of freedom and that
ii) the component coordinate equations of motion for superstring (121), (122) are satisfied.

Let us consider the transformations

$$\delta P_I^M = 2(-)^N \mathcal{D}_N S^{NMI}. \tag{125}$$

where $\Sigma^{KMN} = \Sigma^{[KMN\}}(\zeta)$ is a graded–antisymmetric supertensor and

$$\mathcal{D}S^{NMI} \equiv dS^{NMI} + S^{NMI}A^{JI} = d\zeta^K \mathcal{D}_K S^{NMI}. \tag{126}$$

The variation of the action (55) is

$$\delta_2 S = \int d^2\xi \hat{d}^8\eta 2(-)^N \mathcal{D}_N S^{NMI} E_M^I = \tag{127}$$

$$\int d^2\xi \hat{d}^8\eta (4iS^{MNI}\gamma^I_{q\dot{q}} E_{Nq}^{\ +} E_{M\dot{q}}^{\ -}(-)^M - S^{MNI}E_N^{++} f_M^{--I} - S^{MNI}E_N^{--} f_M^{++I} =$$

$$\int d^2\xi \hat{d}^8\eta (4iS^{+q+pI}\gamma^I_{q\dot{q}} E_{+q}^{\ M}(-)^M E_{M\dot{q}}^{\ -}(-)^M + \ldots$$

where $\ldots$ denote the terms independent of $S^{+q+pI} = S^{MNI}E_{Nq}^{\ +}E_{Mp}^{\ +}(-)^M$. Thus the gamma-traceless part of S^{+q+pI} indeed disappears from the action variation and thus can be associated with a parameter of a gauge symmetry.

To find dynamical equations and to analyze the dynamical content of the Lagrange multiplier P_I^{+q} let us turn to Eqs. (119), (117). For the sake of simplicity, we turn to the linear approximation where Eqs. (119), (117) acquire the form

$$D_{+q} P_I^{+q} = 2(\eta)^8 T\ h^I, \tag{128}$$

$$P_I^{+q}\gamma^I_{q\dot{q}} = -4T(\eta)^8 \psi_{--\dot{q}}^{\ -} \tag{129}$$

and the gauge symmetry (104) reduces to

$$\delta P_I^{+q} = D_{+p} S^{+p+qI}, \tag{130}$$

In Eqs. (128), (130) D_{+q} is the flat superspace covariant derivative (7) and

$$S^{+p+qI} - S^{+q+pI} = S^{+p+qI}\gamma^I_{q\dot{q}} = \tilde{S}^{+p+pI} = 0.$$

carries **224** irreducible representation of $SO(8)$ (113).

The linearized version of superfield bosonic and fermionic equations (122), (121) is

$$h^I = \propto \partial_{++}\partial_{--} X^I, \qquad \psi_{--\dot{q}}^{\ -} = \propto \partial_{--}\Theta^{-}_{\dot{q}}.$$

Eqs. (102) can be used to write the decomposition of the superfield $\psi_{--\dot{q}}^{\ -}$

$$\psi_{--\dot{q}}^{\ -} = (\psi_{--\dot{q}}^{\ -})_0(\xi) - \frac{1}{2}\gamma^I_{q\dot{q}} h_0^I + \ldots \tag{131}$$

To proceed further, the following identities are useful (see Eqs. (5) for definitions)

$$D_{+q}(\eta)^8 = (\eta)^7_q, \quad D_{+p}(\eta)^7_q = (\eta)^6_{qp} + 2i(\eta)^8\delta_{qp}\partial_{++}, \quad D_{+q}(\eta)^7_q = 16i(\eta)^8\partial_{++}, \tag{132}$$

The general solution of Eq. (129) has the form

$$P_I^{+q} = \tilde{P}_I^{+q} - \frac{1}{2}(\eta)^8 \, T \, \psi_{--\bar{\dot{q}}}, \tag{133}$$

where $\tilde{P}_I^{+q}$ is gamma–traceless

$$\tilde{P}_I^{+q}\gamma^I_{q\dot{q}} = 0. \tag{134}$$

Substituting (133) into Eq. (119) one arrives at the following equation for $\tilde{P}_I^{+q}$

$$D_{+q}\tilde{P}_I^{+q} = \frac{1}{2}(\eta)^7_q\gamma^I_{q\dot{q}}(\psi_{--\bar{\dot{q}}})_0 + 2(\eta)^8 T \, h_0^I$$

However, using the decomposition (131) one can collect two terms of the above equation into the first one, but with the component $(\psi_{--\bar{\dot{q}}})_0$ replaced by the superfield $\psi_{--\bar{\dot{q}}}$.

$$D_{+q}\tilde{P}_I^{+q} = \frac{1}{2}(\eta)^7_q\gamma^I_{q\dot{q}}\psi_{--\bar{\dot{q}}} \tag{135}$$

The general solution of Eqs. (135) for *gamma-traceless* $\tilde{P}_I^{+q}$ (134) is a sum of the general solution of the homogeneous equation and a particular solution of the inhomogeneous one

$$\tilde{P}_I^{+q} = (\tilde{P}_I^{+q})_{gen} + (\tilde{P}_I^{+q})_{par}, \qquad D_{+q}(\tilde{P}_I^{+q})_{gen} = 0.$$

The former has the form

$$(\tilde{P}_I^{+q})_{gen} = D_{+p}\Sigma^{+p+qI},$$

with the parameter Σ^{+p+qI} satisfying

$$\Sigma^{+p+qI} - \Sigma^{+q+pI} = \Sigma^{+p+qI}\gamma^I_{q\dot{q}} = \Sigma^{+p+pI} = 0.$$

It can be gauged away by the symmetry (130). Thus the solution of the equation (135) does not contain any indefinite constants. Hence, just at this point, one can conclude that the Lagrange multiplier P_I^M does not contain any dynamical degree of freedom.

The general solution of (135) thus reduces to a particular solution. The latter can be easily obtained using the decomposition of the superfield in the components

$$\tilde{P}_I^{+q} = -\frac{i}{2}(\eta)^7_q T\frac{1}{\partial_{++}}h_0^I + \frac{1}{2}(\eta)^8 T\gamma^I_{q\dot{q}}(\psi_{--\bar{\dot{q}}})_0 \tag{136}$$

However, the solution is not traceless. When we substitute (136) in the tracelessness conditions (134), we find that

$$h_0^I = 0, \qquad (\psi_{--\bar{\dot{q}}})_0 = 0, \qquad \tilde{P}_I^{+q} = 0.$$

In other words, there is no a solution of Eq. (135) with traceless P_I^{+q}.

Thus the Lagrange multiplier P_I^M can be gauged away and the component bosonic and fermionic coordinate equations (122), (121) emerge as a result of Eqs. (119), (117).

Note that local $n = (8,0)$ worldsheet supersymmetry requires that if the leading component of a superfield is equal to zero, then the whole superfield is equal to zero as well (cf. [18]). Thus the *supersymmetric* solution of Eqs. (119), (117) corresponds to superfield equations (48), (49), which are produced by the generalization action.

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